

PHI 169 – CRITICAL REASONING - SPRING 2018 – PROBABILITY EXAM #1

Consider the statistics by the Federal Bureau of Justice Statistics. As convenient notation, let WM , BM and HM stand for “a person is a white male”, “a person is a black male” and “a person is a hispanic male”, respectively. Let I stand for “a person, male or female, is incarcerated”. Calculate the following probabilities or answer the following questions:

- (a) $P(I)$ in 2001 and $P(I)$ in 1974. Did the probability of being incarcerated increase or decrease? If yes, by how much?
(NB: Look up on-line the total US population in 2001 and 1974 to answer this question.)
- (b) $P(WM|I)$ in 2001.
- (c) $P(BM|I)$ in 2001.
- (d) If one is incarcerated in 2001, is this person more probably a white male or a black male?
- (e) What are people saying—exactly—when they say that blacks are disproportionately represented in the incarcerated population? Express this using the language of conditional probabilities. *Hint*: think about the difference between $P(I|BM)$ and $P(BM|I)$ and compare them with $P(I|WM)$ and $P(WM|I)$.

SOLUTIONS

(a) By looking up on Google, one finds:

- total US population in 1974 was 213.9 million
- total US population in 2001 was 285 million

By looking at the statistics from FBJIS, one finds:

- US population incarcerated in 1974 was 1.819 million
- US population incarcerated in 2001 was 5.618 million

The required probabilities can now be calculated, keeping in mind that $P = (\dots) = \frac{PART}{WHOLE}$

- in 1974 $P(I) = \frac{1.819}{213.9} = 0.0085039731 \approx 0.8\%$

- in 2001 $P(I) = \frac{5.618}{285} = 0.0197122807 \approx 2\%$

The probability increased by a factor of $\frac{0.02}{0.008} = 2.5$

(b) The rule is $P(A|B) = \frac{\# \text{ elements in } A \cap B}{\# \text{ elements in } B}$. Our B is I (“a person, male or female, is incarcerated”). Similarly, $A \cap B$ in this case is $WM \cap I$. So, we have

$$P(WM|I) = \frac{\# \text{ elements in } WM \cap I}{\# \text{ elements in } I}.$$

We know from the FBJIS that in 2001 the set I counts 5.618 million people and set $WM \cap I$ counts 1.978 million. So $P(WM|I) = \frac{\# \text{ elements in } WM \cap I}{\# \text{ elements in } I} = \frac{1.978}{5.618} \approx 0.35 \approx 35\%$.

(c) Follow the same procedure but replace WM with BM . From the FBJIS we know that in 2001 black males incarcerated were 1.936 million. So, $P(BM|I) = \frac{1.936}{5.618} \approx 0.34 \approx 34\%$.

(d) The person is more likely to be white male even though by a slight margin. By the calculations in (b) and (c), we discovered that $P(BM|I) \approx 34\%$ and $P(WM|I) \approx 35\%$.

(e) The claim about racial disproportion is $P(I|BM) > P(I|WM)$, although $P(BM|I) < P(WM|I)$.

Consider US Census from 2000. The US population was 281,421,906 and blacks make up 12%, that is, approximately 34 million. Half should be black males, that is, approximately 17 million. So, $P(I|BM) = \frac{\# \text{ elements in } I \cap BM}{\# \text{ elements in } BM} = \frac{1.936}{17} = 0.11388235294 \approx 11\%$

Non-hispanic white in 2000 were roughly roughly 70% of the US population, that is, approximately, 195 million. Half should be white males, that is, approximately 97.5 million.

$$\text{So, } P(I|WM) = \frac{\# \text{ elements in } WM \cap I}{\# \text{ elements in } WM} = \frac{1.978}{97.5} = 0.02028717948 \approx 2\%$$

Black males were more likely to be incarcerated than white males by a factor of $\frac{11}{2} = 5.5$. This is what is meant by racial disproportion among incarcerated people.